EXPERIMENT 14
TORSION CONSTANT OF A ROD

I. THEORY

The purpose of this experiment is to measure the torsion constant of a rod by two methods.

The torsion constant $K$ of a rod is defined by the equation

$$K = \frac{\tau}{\theta}.$$  

in which a torque $\tau$ causes one end of a rod to rotate through an angle $\theta$, measured in radians, while the other end of the rod is fixed. The torsion constant has units of N-m/rad in the SI system.

In order to measure the torsion constant by static means, we clamp one end of the rod in a fixed support, and attach the other end of the rod to a wheel of radius $R$, whose axis of rotation is horizontal. Then we hang a mass $m$ from a strip of flexible metal which wraps around the wheel. The torque exerted is $mgR$.

We then measure the angle $\theta$ through which the wheel rotates. This measurement is made for several values of $m$. From its definition, we see that the torsion constant may be obtained by plotting a graph of torque vs. angular displacement and calculating the slope.

The torsion constant of the rod may be measured dynamically by setting up a torsion pendulum; that is, by attaching the upper end of the rod to a fixed support, and attaching a system with rotational inertia $I$ to the lower end. Such a pendulum oscillates by twisting the rod, not by bending it. The theoretical period of the torsion pendulum is given by

$$T = 2\pi \sqrt{\frac{I}{K}}.$$  

We will consider three systems: a single disk, the disk with a ring placed on it and the disk with two cylinders placed with outer edges tangent to the disk. In all cases, the moment of inertia of the rod itself will be ignored. This is justified since its radius and its mass are both much less than those of the disk.
II. LABORATORY PROCEDURE

The STATIC METHOD and DYNAMIC METHOD can be performed in either order, but you must use the same rod in both parts of the experiment. Record the number of the rod used.

STATIC METHOD

1. Clamp the rod in the static torsion device so that the thumbscrews fit into the V-shaped grooves at both ends of the rod. Wind the metal wire around the wheel. Hang a weight hanger from the end of the wire. Rotate the vernier scale on the apparatus to obtain a reading of 0.0°.

2. Add sufficient masses to the rod to twist the rod between 10° and 13°. Record the angle, reading the vernier scale to the nearest 0.1°, and record the mass added to the weight hanger.

3. Let the mass of Step 2 be called M. Record the angle and the mass on the weight hanger for masses of 2M, 3M, …, 6M.

4. Use the large calipers and a meter stick to measure the diameter of the wheel. Remove the rod from the static torsion device.

DYNAMIC METHOD

1. Attach the rod securely to the wall-mounted brackets, making sure that the thumbscrew fits into the V-shaped groove. Attach the disk securely to the lower end of the rod, again making sure that the thumbscrew fits into the groove.

2. Rotate the disk through an angle of about 10° and release it. Using a stop watch, measure the time for 50 oscillations. Start the stopwatch as the disk comes momentarily to rest at one extreme position. Record the measured time. Carry out a second run.

3. Place the two tall cylinders or two short cylinders on the disk. They should be opposite each other and tangent to the disk. Carry out two runs as in step 6.

4. Replace the cylinders by the ring and carry out two runs.

5. Record the mass of the disk and the ring, which are stamped on them.

6. Place one cylinder on the left pan of the large double-pan balance. Using standard masses and the sliding mass, determine the mass of the cylinder to the nearest gram. Repeat this for the other cylinder and record both masses.

7. Using calipers and a meter stick, measure and record the diameters of the disk and cylinders. Measure and record the inner and outer diameters of the ring.
III. CALCULATIONS

1. Using the data taken with the static torsion device, make a table containing the columns Angular Displacement (deg), Angular Displacement (rad), Mass Added to the Weight Hanger (g) and Torque Applied to the Rod (Nm). Show the calculation of the torque in one case. The torque calculated should just be the torque exerted by the mass added to the weight hanger. Ignoring the torque applied by the weight hanger will have no effect on the slope of the graph plotted next; it would only affect the intercept.

2. Plot a graph of the Applied Torque vs. the Angular Displacement. Use a straightedge to draw the straight line which best fits the plotted points. Calculate the slope of the line.

3. For the torsion pendulum, make a table with the following column headings: System, Average Experimental Period, Moment of Inertia, and Calculated Torsion Constant. The "Systems" are the Disk Alone, the Disk and Ring, and the Disk and Cylinders. Include the results of the following three steps in the table.

4. Calculate the average experimental period for each of the three systems.

5. Calculate the moment of inertia of the disk and the moment of inertia of the ring. Calculate the total moment of inertia of each system. Show your work and remember that you will need to use the Parallel Axis Theorem for the cylinders placed on the disk.

6. Using the equation for the theoretical period of the torsion pendulum, calculate the torsion constant of the rod from the data for each system. Also, calculate the average of these three values and find the percent difference between this average and the slope of the graph.